

Proofs

[1] Fuzzy eqn's For De Morgan

$$a) 1 - \max[\mu_A(x), \mu_B(x)] = \min[1 - \mu_A(x), 1 - \mu_B(x)]$$

Sol

assume :- $\mu_A(x) \leq \mu_B(x)$

$$L.H.S = 1 - \max[\mu_A(x), \mu_B(x)] = 1 - \mu_B(x)$$

$$\mu_A(x) \leq \mu_B(x) \Rightarrow -\mu_A(x) \geq -\mu_B(x)$$

$$1 - \mu_A(x) \geq 1 - \mu_B(x)$$

$$R.H.S = \min[1 - \mu_A(x), 1 - \mu_B(x)]$$

$$= 1 - \mu_B(x) = L.H.S$$

$$\therefore L.H.S = R.H.S$$

هو نفسه القانون الآخر

$$(A \cup B)^c = A^c \cap B^c$$

b)

$$1 - \min[M_A(x), M_B(x)] = \max[1 - M_A(x), 1 - M_B(x)]$$

Let: $M_A(x) \leq M_B(x)$

$$\text{L.H.S} = 1 - \min[M_A(x), M_B(x)]$$

$$\text{L.H.S} = 1 - M_A(x)$$

$$-M_A(x) \geq -M_B(x) \Rightarrow 1 - M_A(x) \geq 1 - M_B(x)$$

$$\therefore \max[1 - M_A(x), 1 - M_B(x)] = \text{R.H.S}$$

$$\text{R.H.S} = 1 - M_A(x)$$

$$\therefore \text{R.H.S} = \text{L.H.S} \quad \text{///}$$

ده نفسه القانين

$$(A \cap B)^c = A^c \cup B^c$$

2] Show that Yager Fuzzy Complement and Sugeno satisfies Complement Axioms.

Axioms are:-

$$1] c(1) = 0, c(0) = 1$$

$$2] a = \mu_A(x) ; b = \mu_B(x)$$

$$a \leq b \Rightarrow c(a) \geq c(b)$$

A) Sugeno

$$C_{\lambda}(a) = \frac{1-a}{1+\lambda a} ; a = \mu_A(x)$$

$$c(0) = \frac{1-0}{1+0} = 1 ; c(1) = \frac{1-1}{1+\lambda} = 0 \quad \#$$

$$a \leq b \Rightarrow \lambda a \leq \lambda b ; \lambda \geq 0$$

$$\therefore 1 + \lambda a \leq 1 + \lambda b$$

$$\frac{1}{1+\lambda a} \geq \frac{1}{1+\lambda b} \rightarrow (1)$$

$$a \leq b \Rightarrow -a \geq -b \Rightarrow 1-a \geq 1-b \rightarrow (2)$$

Multiply (1) & (2)

$$\frac{1-a}{1+\lambda a} \nlessgtr \frac{1-b}{1+\lambda b} \Rightarrow C(a) \nlessgtr C(b) \neq$$

→ Sugeno satisfy Complement axioms.

B) Yager

$$C_w(a) = (1 - a^w)^{\frac{1}{w}}$$

$$C_w(0) = (1 - 0^w)^{\frac{1}{w}} = 1 \quad ; \quad C_w(1) = (1 - 1^w)^{\frac{1}{w}} = 0$$

$$a \leq b \Rightarrow C_w(a) \nlessgtr C_w(b)$$

$$a \leq b \rightarrow a^w \leq b^w \Rightarrow 1 - a^w \nlessgtr 1 - b^w$$

$$1 - a^w \nlessgtr 1 - b^w \Rightarrow (1 - a^w)^{\frac{1}{w}} \nlessgtr (1 - b^w)^{\frac{1}{w}}$$

$$\therefore C_w(a) \nlessgtr C_w(b)$$

⇒ Yager satisfy Complement axioms.

3] Show that Dombi union operation satisfy s-norm axioms.

Dombi Class:

$$S_{\lambda}(a, b) = \frac{1}{1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{b} - 1 \right)^{-\lambda} \right]^{\frac{1}{\lambda}}}$$

\square $a = \mu_A(x)$, $b = \mu_B(x)$

\square $S(1, 1) = 1$, $S(0, a) = S(a, 0) = a$

$S(1, 1) = 1$

$$S(0, a) = \frac{1}{1 + \left[0 + \left(\frac{1}{a} - 1 \right)^{-\lambda} \right]^{\frac{1}{\lambda}}}$$

$$= \frac{1}{1 + \left(\frac{1}{a} - 1 \right)} = \boxed{a}$$

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$$s(a, 0) = \frac{1}{1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} \right]^{\frac{-1}{\lambda}}} = a$$

$$\therefore s(a, 0) = s(0, a) = a \quad \# \quad [1]$$

$$[2] \quad s(a, b) = s(b, a)$$

$$s(a, b) = \frac{1}{1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{b} - 1 \right)^{-\lambda} \right]^{\frac{-1}{\lambda}}}$$

$$= \frac{1}{1 + \left[\left(\frac{1}{b} - 1 \right)^{-\lambda} + \left(\frac{1}{a} - 1 \right)^{-\lambda} \right]^{\frac{-1}{\lambda}}} = s(b, a) \quad \# \quad [2]$$

$$[3] \quad a \leq \bar{a}, \quad b \leq \bar{b}$$

$$\therefore s(a, b) \leq s(\bar{a}, \bar{b})$$

$$a \leq \bar{a} \Rightarrow \frac{1}{a} \geq \frac{1}{\bar{a}}$$

$$\frac{1}{a} - 1 \geq \frac{1}{\bar{a}} - 1 \Rightarrow \left(\frac{1}{a} - 1 \right)^{\lambda} \geq \left(\frac{1}{\bar{a}} - 1 \right)^{\lambda}$$

$$\left(\frac{1}{a} - 1\right)^{-\lambda} \leq \left(\frac{1}{a} - 1\right)^{-\lambda}$$

$$\underline{\underline{\text{مثال}}} \quad \left(\frac{1}{b} - 1\right)^{-\lambda} \leq \left(\frac{1}{b} - 1\right)^{-\lambda}$$

$$\therefore \left(\frac{1}{a} - 1\right)^{-\lambda} + \left(\frac{1}{b} - 1\right)^{-\lambda} \leq \left(\frac{1}{a} - 1\right)^{-\lambda} + \left(\frac{1}{b} - 1\right)^{-\lambda}$$

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$$\left[\begin{array}{c} * \end{array} \right]^{-\frac{1}{\lambda}} \geq \left[\begin{array}{cc} * & * \end{array} \right]^{-\frac{1}{\lambda}}$$

$$1 + \left[\begin{array}{c} * \end{array} \right]^{-\frac{1}{\lambda}} \geq \left[\begin{array}{cc} * & * \end{array} \right]^{-\frac{1}{\lambda}}$$

$$\frac{1}{1 + \left[\begin{array}{c} * \end{array} \right]^{-\frac{1}{\lambda}}} \leq \frac{1}{1 + \left[\begin{array}{cc} * & * \end{array} \right]^{-\frac{1}{\lambda}}}$$

$$\Rightarrow S(a, b) \leq S(\tilde{a}, \tilde{b}) \neq [3]$$

$$\boxed{A} \quad S(S(a,b),c) = S(a, S(b,c)) \quad ?$$

$$L.H.S = S(S(a,b),c) = \frac{1}{1 + \left[\left(\frac{1}{S(a,b)} - 1 \right)^{-\lambda} + \left(\frac{1}{c} - 1 \right)^{-\lambda} \right]^{\frac{1}{\lambda}}}$$

$$\frac{1}{S(a,b)} = 1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{b} - 1 \right)^{-\lambda} \right]^{\frac{1}{\lambda}}$$

$$\left(\frac{1}{S(a,b)} - 1 \right)^{-\lambda} = \left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{b} - 1 \right)^{-\lambda}$$

$$L.H.S = \frac{1}{1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{b} - 1 \right)^{-\lambda} + \left(\frac{1}{c} - 1 \right)^{-\lambda} \right]^{\frac{1}{\lambda}}}$$

$$R.H.S = S(a, S(b,c))$$

$$= \frac{1}{1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{S(b,c)} - 1 \right)^{-\lambda} \right]^{\frac{1}{\lambda}}}$$

$$s(b, c) = \frac{1}{1 + \left[\left(\frac{1}{b} - 1 \right)^{-\lambda} + \left(\frac{1}{c} - 1 \right)^{-\lambda} \right]^{\frac{-1}{\lambda}}}$$

$$\frac{1}{s(b, c)} = 1 + \left[\left(\frac{1}{b} - 1 \right)^{-\lambda} + \left(\frac{1}{c} - 1 \right)^{-\lambda} \right]^{\frac{-1}{\lambda}}$$

$$\left(\frac{1}{s(b, c)} - 1 \right)^{-\lambda} = \left(\frac{1}{b} - 1 \right)^{-\lambda} + \left(\frac{1}{c} - 1 \right)^{-\lambda}$$

$$R.H.S = \frac{1}{1 + \left[\left(\frac{1}{a} - 1 \right)^{-\lambda} + \left(\frac{1}{b} - 1 \right)^{-\lambda} + \left(\frac{1}{c} - 1 \right)^{-\lambda} \right]^{\frac{-1}{\lambda}}}$$

$$R.H.S = L.H.S \quad \neq [4]$$

→ Dombi union satisfy s-norm axioms.

Convex

مسائل من السهولة على 1 قانون 1)

[4] Convex

$$M[\lambda x_1 + (1-\lambda)x_2] \geq \min[M_{x_1}, M_{x_2}]$$

For example: $M = \frac{1}{1+x^2}$

Let $M_{x_1} < M_{x_2}$

R.H.S = $\min(M_{x_1}, M_{x_2}) = M_{x_1} = \frac{1}{1+x_1^2} \rightarrow (1)$

L.H.S = $M(\lambda x_1 + (1-\lambda)x_2)$

$$= \frac{1}{1 + [\lambda x_1 + (1-\lambda)x_2]^2} = \frac{1}{1 + (\lambda x_1 + x_2 - \lambda x_2)^2}$$

Replace $x_2 \rightarrow x_1$

$$L.H.S = \frac{1}{1 + (\lambda x_1 + x_1 - \lambda x_1)^2} = \frac{1}{1 - x_1^2} \rightarrow (2)$$

L.H.S = R.H.S $\therefore M \rightarrow$ is Convex

[2] $x \leq 10$

$$\mu = \begin{cases} 0 & x \leq 10 \\ \frac{1}{1 + (x - 10)^{-2}} & x > 10 \end{cases}$$

[Sol]

at $\mu = 0$ $\mu_{x_1} < \mu_{x_2}$

R.H.S = $\min [\mu_{x_1}, \mu_{x_2}] = \mu_{x_1} = 0$

L.H.S = $\mu [\lambda x_1 + (1 - \lambda)x_2] = 0 \rightarrow \text{Convex}$

For $\mu = \frac{1}{1 + (x - 10)^{-2}}$

$\mu_{x_1} < \mu_{x_2}$

R.H.S = $\min [\mu_{x_1}, \mu_{x_2}] = \mu_{x_1} = \frac{1}{1 + (x_1 - 10)^{-2}}$

$\mu_T(x)$

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$$\underline{\underline{L-H.S}} = \mathcal{M} [\lambda x_1 + (1-\lambda)x_2]$$

$$= \frac{1}{1 + (\lambda x_1 + (1-\lambda)x_2 - 10)^{-2}} \quad \text{replace } x_2 \rightarrow x_1$$

$$= \frac{1}{1 + (x_1 - 10)^{-2}} = R-H.S \quad \therefore \text{Convex}$$